- **5283:** *Proposed by Kenneth Korbin, New York, NY*

  Find the sides of two different isosceles triangles that both have perimeter 162 and area 1008.

Solution 2 by Arkady Alt, San Jose, CA

Let b be length of the lateral sides and a be half of length of the base.

Then $\begin{cases} 2a+2b=162 \\ a\sqrt{b^2-a^2}=1008 \end{cases} \iff \begin{cases} a+b=81 \\ a\sqrt{b-a}=112 \end{cases} \iff \begin{cases} b=81-a \\ a\sqrt{81-2a}=112 \end{cases}$

We have $a\sqrt{81-2a}=112 \iff \begin{cases} 0<a\le 81/2 \\ a^2(81-2a)=112^2 \end{cases}$ and the equation

$a^2(81-2a)=16^2\cdot 49 \iff 2a^3-81a^2+112^2=0.$

Since $2a^3-81a^2+112^2=(a-16)\left(2a^2-49a-784\right)$ and the quadratic equation

$2a^2-49a-784=0$ have only one positive root $a=\dfrac{49+7\sqrt{177}}{4}$ then we obtain two different isosceles triangles with side-lengths

$(b,2a,b)=(65,32,65)\,,\quad \left(\dfrac{275-7\sqrt{177}}{4},\dfrac{49+7\sqrt{177}}{2},\dfrac{275-7\sqrt{177}}{4}\right).$